Rotational Dynamics Examples

① (a) $\omega_i = 32.0 \text{ rad s}^{-1}$

$\omega_f = 0$

$\alpha = -0.7 \text{ rad s}^{-2}$

$t = ?$

$\omega_f = \omega_i + \alpha t$

$t = \dfrac{\omega_f - \omega_i}{\alpha} = \dfrac{-32.0}{-.7} = \underline{45.7 \text{ s}}$

(b) $\theta = ?$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$\theta = \dfrac{-\omega_i^2}{2\alpha} = \dfrac{-(32.0)^2}{2(-.7)} = 731.429 \text{ rad.}$

revolutions $\dfrac{\theta}{2\pi} = \dfrac{731.429}{2\pi} = \underline{116}$

② $\Gamma = I\alpha \qquad \Gamma = Fr \sin\theta \qquad \theta = 90$

$Fr = I\alpha$

$I = \dfrac{Fr}{\alpha} = \dfrac{(2\times10^3 \text{ N})(3\times10^{-2} \text{ m})}{120 \text{ rad s}^{-2}} = \underline{0.50 \text{ Nm rad}^{-1} \text{s}^{-2}}$

(3) (a) $\Gamma = Fr \sin\theta$

$= (180N)(0.28m)$

$= 50.4 = \underline{50\ Nm}$

(b) $\Gamma = I\alpha$

$\alpha = \frac{\Gamma}{I} = \frac{\Gamma}{\frac{1}{2}MR^2} = \frac{50.4\ Nm}{\frac{1}{2}(75kg)(.28m)^2} = 17.14 = \underline{17.1\ rad\ s^{-1}}$

(c) $\Gamma = Fr_1 + F_f r_2$

$= 180(.28) - 20(1.5 \times 10^{-2}\ m) = 50.1\ Nm$

$\alpha = \frac{F}{\frac{1}{2}MR^2} = \frac{50.1}{\frac{1}{2}(75)(.28)^2} = 17.04 = \underline{17.0\ rads^{-1}}$

(4) $\Gamma_{cw} = \Gamma_{ccw}$

$mgL = T \sin\theta\, x$

$T = \frac{mgL}{\sin\theta\, x}$

(5) $L_{before} = L_{after}$ $\quad \omega = 2\pi f$

$I_{before}\,\omega_b = I_a\,\omega_a$

$$\left(\tfrac{1}{2}MR^2 + m_{22}r_{22}^2 + m_{28}r_{28}^2 + m_{33}r_{33}^2\right) 2\pi f_b =$$

$$\left(\tfrac{1}{2}MR^2 + m_{22}r_{22}^2 + m_{33}r_{33}^2\right) 2\pi f_a$$

$$\left(\tfrac{1}{2}m + m_{22} + m_{28} + m_{33}\right) R^2 f_b = \left(\tfrac{1}{2}M + m_{22} + m_{33}\right) R^2 f_a$$

$$f_a = \frac{\left(\frac{1}{2}m + m_{22} + m_{28} + m_{33}\right) f_b}{\left(\frac{1}{2}m + m_{22} + m_{33}\right)}$$

$$= \frac{\left(\frac{1}{2}(100) + 22 + 28 + 33\right)\left(\frac{20}{60}\right)}{\left(\frac{1}{2}(100) + 22 + 33\right)}$$

$$= 0.4222\ \text{Hz} \times 60$$

$$f_a = 25\ \text{rpm}$$

(6) (a) $L_{before} = L_{after}$ $\omega = 2\pi f = \frac{2\pi}{T}$

$I_b \omega_b = I_a \omega_a$

$\frac{1}{2} m R_1^2 \frac{2\pi}{T_b} = \frac{1}{2} m R_2^2 \frac{2\pi}{T_a}$ $R_1 = 10000\, R_2$

$(10000)^2 \frac{R_2^2}{T_b} = \frac{R_2^2}{T_a}$

$T_a = \frac{T_b}{(10000)^2} = \frac{T_b}{1\times10^8}$

$T_a = \frac{20\times24\times3600}{1\times10^8} = 0.017 = \underline{0.025}$

(b) $E_{k\,rot} = \frac{1}{2} I \omega^2$

$\frac{E_{k\,rot\,a}}{E_{k\,rot\,b}} = \frac{\frac{1}{2}\left(\frac{1}{2} m R_2^2\right)\left(\frac{2\pi}{T_a}\right)^2}{\frac{1}{2}\left(\frac{1}{2} m R_1^2\right)\left(\frac{2\pi}{T_b}\right)^2} = \frac{R_2^2 \left(\frac{1}{T_a}\right)^2}{(10000)^2 R_2^2 \left(\frac{1}{T_b}\right)^2}$

$= \frac{T_b^2}{(1\times10^8) T_a^2} = \frac{T_b^2 (1\times10^8)^2}{(1\times10^8) T_b^2}$

$E_{k\,rot}\ \frac{after}{before} = 1\times10^8$ $E_{k\,rot}\ \frac{before}{after} = 1\times10^{-8}$

(7) $E_{k_{rot}} = \frac{1}{2} I \omega^2$ $\qquad v = \omega r$

$= \frac{1}{2} I \frac{v^2}{r^2}$

$= \frac{1}{2}(.5)\frac{(20)^2}{(.48)^2}$

$= \underline{434 \text{ J}}$

(8) $E_k = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$

$= \frac{1}{2}\left(\frac{1}{2} m r^2\right)\frac{v^2}{r^2} + \frac{1}{2} m v^2$

$= \frac{1}{4} m v^2 + \frac{1}{2} m v^2$

$E_k = \underline{\frac{3}{4} m v^2}$

(9) (a) $L = I\omega$ $\qquad \omega = 2\pi f$

$L = I 2\pi f$

$= .4(2\pi)6$

$= 15.08 = \underline{15.1 \text{ kg m}^2 \text{ rad s}^{-1}}$

(b) $L_b = L_a$

$I_b 2\pi f_b = I_a 2\pi f_a$

$I_a = \dfrac{I_b f_b}{f_a}$

$= \dfrac{(.4)6}{1.25}$

$\underline{I = 1.92 \text{ kg m}^2}$

(c) $\Gamma = I\alpha$ $\qquad \omega_f = \omega_i + \alpha t$

$= \dfrac{I 2\pi (f_f - f_i)}{t}$ $\qquad \alpha = \dfrac{\omega_f - \omega_i}{t}$

$= \dfrac{2\pi(f_f - f_i)}{t}$

$= \dfrac{.4(2\pi)(3-6)}{15}$

$\Gamma = 0.503$ Nm in the opposite direction of the spinning.